INTERVALS

Intervals are basically subsets of R and are of very much importance in calculus as you will get to know shortly Intervals are particularly important when you are asked to solve inequalities or to find domains etc. If there are two numbers a, b ∈ R such that a < b. we can define four types of intervals as follows:

- Open Interval: (a, b) = {x : a < x < b} i.e. end points are not included.

- Closed Interval: [a,b]= {x : a ≤ x ≤ b} i.e. end points are also included. This is possible only when both a and b are finite.

- Open-closed Interval : (a, b] ={x : a< x ≤ b}

- Closed-open Interval [a, b)= {x : a≤ x< b}

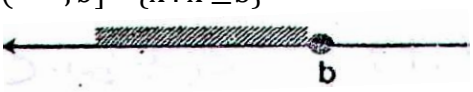
The infinite or unbounded intervals are defined as follows:

- (a, ∞)= {x : x >a}

- [a, ∞)= {x : x ≥a}

- $(-\infty, b)$= {x : x <b}

- $(-\infty, b]$= {x : x ≤b}



Clearly the interval $(-\infty, \infty)$ = {x : −∞ <x <∞} represents the entire real number line, hence the set R itself.

BASIC INEQUALITIES

The following are some very useful points to remember:

- $a \leq b \Rightarrow$ either $a < b$ or $a = b$
- $a < b$ and $b < c \Rightarrow a < c$
- $a < b \Rightarrow a + c < b + c \; \forall \; c \in R$
- $a < b$ and $c < d \Rightarrow a + c < b + d$ and $a - d < b - c$
- $a < b \Rightarrow ka < kb$ if $k > 0$ and $ka > kb$ if $k < 0$. i.e. inequality sign reverses if both sides are multiplied by a negative number, in particular $a < b \Rightarrow -a > -b$.
- $0 < a < b \Rightarrow a^r < b^r$ if $r > 0$ and $a^r > b^r$ if $r < 0$
- $\left(f(x) + \frac{1}{f(x)}\right) \geq 2 \; \forall \; f(x) > 0$ and equality holds for $f(x) = 1$.
- $\left(f(x) + \frac{1}{f(x)}\right) \leq -2 \; \forall \; f(x) < 0$ and equality holds for $f(x) = -1$.

LOGARITHM:

- Following are some useful points to remember:
- The expression $\log_b x$ is meaningful for $x > 0$, $b > 0$ and $b \neq 1$
- $b^{\log_b a} = a$
- $\log_a b = \frac{\log_c b}{\log_c a}$
- $\log_b a = \frac{1}{\log_a b}$ provided both a and b are non-unity.
- $\log_b a_1 \geq \log_b a_2 \Rightarrow \begin{cases} a_1 \geq a_2 > 0 & if \; b > 1 \\ 0 < a_1 \leq a_2 & if \; 0 < b < 1 \end{cases}$

FUNDAMENTAL OF MATHEMATICS [1]

1. The value of $\sqrt{36}$ is equal to
a) ± 6 b) $+6$ c) -6 d) None of these

2. The value of'$\sqrt{a^2-4a+4}$ is equal to
a)$\pm$ |a−2| b) |a−2| c)2−a d)None of these

3. If $x^2 = 36$ then x equals to
a) ± 6 b) $+6$ c) -6 d) None of these

4. If |2x− 3|=2x−3 then
a) $x \leq \frac{3}{2}$ b) $x \geq \frac{3}{2}$ c) $x < \frac{3}{2}$ d) $x > \frac{3}{2}$

5. If |x−1| >0 then
a) $x \in R-\{1\}$ b) $x \in R$ c) $x \in R^+$ d)$x \in R^-$

6. If $|x^2 - 1| + (x-1)^2 + \sqrt{x^2-3x+2} = 0$ then
a) $x = \pm 1$ b) $x = 1, 2$ c) x=−1,2 d) $x = 1$

7. If $4^x - 2^x - 2 = 0$ then
a) x =−1, 2 b) $x = -1$ c) x=2 d) x=1

8. If $\sqrt{x^2} = -x$ then
a) $x = R^+$ b) $x = R^-$ c) x =0 d) None of these

9. If |x| > x then
a) $x < 0$ b) $x \leq 0$ c) $x > 0$ d) $x \geq 0$

10. If $\frac{(2x-1)(x-1)^2(x-2)^2}{(x-2)(x-4)^4} > 0$ then
a) $x \in (-\infty, \frac{1}{2}) \cup (2, \infty)$ b) $x \in (-\infty, \frac{1}{2}) \cup (2, \infty) \cup (4, \infty)$
c) $x \in (-\infty, \frac{1}{2}] \cup [2, \infty)$ d) $x \in (-\infty, \frac{1}{2}] \cup [2, 4)] \cup [4, \infty)$

11. The L.C.M of 2π and 1 is
a) 2π b) π c) 1 d) L.CM is not defined.

12. The L.C.M of $\sqrt{2}$ and $\sqrt{3}$ is
a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\sqrt{2} \times \sqrt{3}$ d) LC.M doesn't exist.

13. The values of x for which $f(x) = (x-2)^2 (1-x) (x-3)^3 (x-4)^{100} \leq 0$ is
a) $-\infty < x \leq 1$ or $3 \leq x \leq 4$ or $4 \leq x \leq \infty$ or $x = 2$
b) $-\infty < x \leq 1$ or $3 \leq x \leq 4$ or $4 \leq x < \infty$
c) $-\infty < x \leq 1$ or $3 \leq x \leq 4$
d) None of these

14. If $|x-1| + |x-3| = 2$ then
a) $1 < x \leq 3$ b) $1 \leq x \leq 3$ c) $x > 3$ d) none of these

15. If $x^2 - 7x + 12 > 0$ then
a) $x \in (-\infty, 3) \cup (4, \infty)$ b) $x \in (3, 4)$
c) $x \in (-\infty, 3) \cup (-\infty, 4)$ d) $x \in (3, \infty) \cup (4, \infty)$

16. If $\frac{1-|x|}{2-|x|} \geq 0$ then

a) $(-\infty, \infty) - [-2, 2]$ b) $(-\infty,\infty) - [-1, 1]$

c) $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$ d) None of these

17. The number of positive integral solution of $\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^2(2x-7)^6} \leq 0$ is

a) Four b) Three c) Two d) Only one

18. If $x^2 + x + 1 > 0$ then

a) $x \in R$ b) $x \in R^+$ c) $x \in R^-$ d) None of these

19. If $x^2 + x + 1 < 0$ then

a) $x \in R$ b) $x \in R^+$ c) $x \in R^-$ d) $x = \phi$

20. The number of solution of $\log_4 (x-1) = \log_2 (x-3)$ is

a)only one b)two C) three d)none of these

21. The values of x which satisfies the equation $x^4 - 12x^3 + 41x^2 - 18x - 72 = 0$ are

a) -1 only b) 3,4 only c) $x = -1,3,4$ and 6 d) 3,4 only

22. If $(x+1)^2 + (x^2 + 3x + 2)^2 = 0$ then

a) $x = -1, -2$ b) $x = -1$ c) $x = -2$ d) None of these

23. The values of x for which $\frac{(x-2)^2(1-x)(x-3)^3(x-4)^2}{(x+1)} \leq 0$ are

a) $x \in [-1, 1] \cup [3,\infty)$ b) $x \in (-1, 1) \cup (3,\infty)$ c) $x \in [-2, 2] \cup [3, \infty)$ d) None of these

24. If $p = a^2 - 2ab + b^2$ and $q = 4ab$, then $p + q$ is

a) 2ab b) $(a-b)^2$ c) $(a + b)^2$ d) $a^2 + b^2$

25. $52 \times 48 = (a)^2 - (b)^2$, then the values of a and b are

a) 50, 2 b)52, 48 c)4, 50 d)4, 52

26. The value of $(a-b)(a + b)(a^2 + b^2)(a^4 + b^4)$ when $a=0, b=1$ is

a)0 b)1 c)-1 d)2

27. If $x - \frac{1}{x} = 5$, then $x^3 - \frac{1}{x^3} =$

a) 100 b) 125 c) 140 d) 145

28. If $x + y = 8$; $xy = 12$ then $x^4 + y^4 =$

a) 1012 b) 1112 c) 1212 d) 1312

29. If $z - \frac{1}{z} = 4$ then $z^2 + \frac{1}{z^2} =$ _____

a)18 b)16 c)14 d)8

30. If $x + y + z = 0$ then $x^3 + y^3 + z^3 =$

a) 3 xyz b) xyz c)0 d) 1

31. $(x-y)^3 + (y-z)^3 + (z-x)^3 =$

a) $3(x-y)(y-z)(z-x)$ b) 0 c) 1 d) 3xyz

32. If $a^{\frac{1}{3}}+ b^{\frac{1}{3}}+c^{\frac{1}{3}} = 0$ then $(a + b + c)^3 =$

a) 0 b) 1 c) $(abc)^{\frac{1}{3}}$ d) 27abc

33. The HCF of $2x^2$ and $12x^2$ is

a) $2x^2$ b) $12x^2$ c) 2x d) 12x

34. The HCF of x^3 and $-yx^2$ is

a) x^3 b) $-yx^2$ c) x^2 d) $-yx^5$

35. The HCF of numerical coefficient of the given monomials $6x^3a^2b^2c$, $8x^2ab^3c^3$ and $12a^3b^2c^2$ is

a)12 b)8 c)6 d)2

36. The factorization of $8a^3 - 27$ is

a) $(2a+3)(4a^2 + 6a + 9)$ b) $(2a+3)(4a^2 - 6a + 9)$

c) $(2a-3)(4a^2 + 6a + 9)$ d) $(2a - 3)(4a^2 - 6a + 9)$

37. If $A = 384x^4y^5z^3$ and $B = 256x^2y^3z^3$, then their G. C. D is

a) $2^7x^2y^3z^5$ b) $2^7x^2y^5z^3$ c) $2^7x^2y^3z^3$ d) $2^7x^4y^5z^5$

38. The H.C.F of 15pq, 20qr, 25rp is

a) 5pqr b) 25pqr c)5 d)25

39. The factors of $-10ab^3 + 30ba^3 - 50a^2b^3$ is

a) $-10ab(b^2 - 3a^2 + 5ab^2)$ b) $10ab(b^2 - 3a^2 + 5ab^2)$

c) $-10ab(b^2 + 3a^2 + 5ab^2)$ d)None of these

40. The no. of real roots of the equation $(x-1)^2 + (x- 2)^2 + (x- 3)^2 = 0$ is

a)2 b)1 c)0 d)3

41. If $ax + by + c = 0$ is the general form of the line, then

a) $|a| + |b| = 0$ b) $|a| + |b| \neq 0$ c) $a + b = 0$ d) $a + b \neq 0$

XI - XII Board Pattern:

1. Solve the system of inequalities: $9x^2 - 4$ and $|2x - 1| \leq 3$.

2. Simplify $\sqrt{9 - 6a + a^2} + \sqrt{9 + 6a + a^2}$ for $a < -3$

3. Solve: $|x^2 - 4x + 3| = x^2 - 4x + 3$

4. Solve: $x^2 - 3|x| + 2 = 0$

5. Solve: $x^2 + 3|x| + 2 = 0$

6. Solve: $|x^2 - 4x + 3| = -x^2 + 4x - 3$

7. 0 is an even number. why?

8. If $|x-1| + |x-2| + |x-3| \leq 6$ then find the interval in which x belongs to?

TOPIC: FUNDAMENTAL OF MATHEMATICS [2]

1. If $|2x-3| = 2x-3$ then
a) $x \leq \frac{3}{2}$ b) $x \geq \frac{3}{2}$ c) $x > \frac{3}{2}$ d) $x < \frac{3}{2}$

2. If $|x+1|+\sqrt{x-1} = 0$ then
a) $x=1$ b) $x=-1$ c) $x = \pm 1$ d) No solution

3. Which one is correct relation?
a) $3 \geq 2$ b) $5 > 5$ c) $2<3$ d) none of these

4. The values of x satisfying the inequality $\frac{(2x-1)(x-1)^2(x-2)^2}{(x-2)(x-4)^4} > 0$
a) $-\infty < x < \frac{1}{2} \cup 2 < x < 4 \cup 4 < x < \infty$ b) $-\infty < x < \frac{1}{2} \cup (2, \infty)$
c) $-\infty < x < \frac{1}{2} \cup [2, \infty)$ d) None of these

5. If $\cos x > -\frac{1}{2}$ then
a) $2k\pi - \frac{2\pi}{3} < x < 2k\pi + \frac{2\pi}{3}$ b) $x > \frac{2\pi}{3}$
c) $k\pi - \frac{2\pi}{3} < x < k\pi + \frac{2\pi}{3}$ d) None of these

6. If $\sin x > \cos x$ then
a) $2k\pi - \frac{\pi}{4} < x < 2k\pi + \frac{5\pi}{4}$ b) $2k\pi - \frac{\pi}{4} < x < 2k\pi - \frac{5\pi}{4}$
c) $2k\pi - \frac{\pi}{4} < x < 2k\pi + \frac{5\pi}{4}$ d) None of these

7. The value of $\left[\frac{3}{4}\right]+\left[\frac{3}{4}+\frac{1}{100}\right]+\left[\frac{3}{4}+\frac{1}{100}\right]+\ldots\ldots+\left[\frac{3}{4}+\frac{99}{100}\right] =$
a) 74 b) 75 c) 76 d) 77

8. The real roots of the equation $\log_5 \log_5(x^2-4x+5)+5) = x-1$ are
a) 1, 2 b) 2, 3 c) 3, 4 d) 4 , 5

9. The number of real solutions of $x-\frac{1}{x^2-4} = 2-\frac{1}{x^2-4}$ is
a) 0 b) 1 c) 2 d) infinite

10. The number of solutions of $2^{\sin(|x|)} = 4^{|\cos x|}$ in $[-\pi, \pi]$ is equal to
a) 0 b) 2 c) 4 d) 6

11. The number of positive integral solutions of $\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6} \leq 0$ is
a) Four b) Three c) Two d) Only one

12. If $xy = 2(x+y)$, $x \leq y$ and $x, y \in N$, the number of solution of the equation is
a) two b) three c) no solution d) infinitely many solution

13. The values of a for which the equation $2(\log_3 x)^2 - |\log_3^x| + a = 0$ possess four real solutions
a) $-2<a<0$ b) $0 < a < \frac{1}{8}$ c) $0<a<5$ d) none of these

14. If $0< x< 1000$ and $\left[\frac{x}{2}\right]+\left[\frac{x}{3}\right]+\left[\frac{x}{5}\right]=\frac{31}{30}x$, where [x] is the greatest integer less than or equal to x, the number of possible values of x is

a) 34 b) 33 c) 32 d) none of these

15. The values of $\sqrt{7+\sqrt{7-\sqrt{7+\sqrt{7-\cdots\infty}}}}$ is

a)5 b)4 c)3 d)2

16. If $x^2-5x+6<0$ then

a) $x \in (2,3)$ b) $x<2$ or $x>3$ c) $x \leq 2$ or $x \geq 3$ d) None of these

17. If $x^2+x+1>0$ then

a) $x \in R$ b) $x \in R^+$ c) $x \in R^-$ d) None of these

18. $\frac{0}{0}$

a) 1 b) ∞ c) 0 d) indeterminate

19. $x^{x^x}=$

a) x^{x^2} b) $(x^x)^2$ c) x^{xxx} d) None of these

20. Statement- 1: The curve $y=\frac{x^2}{2}+x+1$ is symmetric with respect to the line $x=1$. Because

Statement – 2 : A parabola is symmetric about its axis

a)…… b)….. c)…….. d) ……..